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Electromagnetic behaviour of superconductive amorphous metals

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Abstract

The penetration depth of the magnetic field into an amorphous superconductor is calculated. The ratio of the London penetration depth δ_L to the electron free path l_e under zero temperature is above unity for almost all amorphous metals. That is why pure metals, in a superconducting state, change from type I superconductors to type II superconductors during the crystalline–amorphous transition.

For all known superconducting metals, including amorphous ones, the essential mechanism responsible for the superconducting state is an electron–phonon interaction. This mechanism and its influence on the different properties of superconductors is described in detail in various monographs [1–5]. However, during the crystal–amorphous state transition of the superconductor, a number of its characteristics change. For example, experimental and theoretical studies of simple amorphous metals show that their transition temperature T_c into a superconducting state is usually bigger than that of crystalline materials [4, 5]. This is connected to an increase of electron–phonon relation constants. This increase occurs due to the breaking of long-range ordering in amorphous metals. As a result, the phonon spectrum softens and diffusive scattering of electrons occurs. The behaviour of amorphous superconductors becomes even more distinct when applying an external magnetic field. Under normal conditions, because of the violation of long-range ordering in the amorphous metal, a residual electrical resistance appears. This causes the electron free path l_e in amorphous metals under zero temperature to be finite. In addition, the electron free path becomes less than the coherence length ξ_0, which appears in between electrons, while transitioning into the superconducting state. As a result, the structural fluctuation scattering of electrons leads to a decrease of the superconducting correlation length. This in turn decreases the penetration depth δ_L of the magnetic field into the superconductor. As a consequence, a crystalline superconductor of type I during its transition into the amorphous state turns into a superconductor of type II.

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Using and generalizing the method developed by Abrikosov and Gorkov, it is possible to use Green temperature functions to investigate the properties of amorphous superconductors in a weak field. This method was created while developing the theory of superconducting crystalline alloys (see [1, 6–9]). In a general case of a spatially unbounded medium, the connection between the Fourier image of the electric current $\mathbf{j}(\mathbf{r})$ and the vector potential of the magnetic field $\mathbf{A}(\mathbf{r})$ into this medium is defined through [1, 2]:

$$\tilde{j}_\alpha(\mathbf{q}) = -Q_{\alpha\beta}(\mathbf{q}) \cdot \tilde{A}_\beta(\mathbf{q}), \tag{1}$$

where the core $Q_{\alpha\beta}(\mathbf{q})$ is written as

$$Q_{\alpha\beta}(\mathbf{q}) = \frac{2\hbar^2 e^2}{m^2 c} \cdot \frac{k_\mathrm{B} T}{(2\pi)^3} \sum_{s=-\infty}^{+\infty} \int \mathrm{d}^3 k \, k_\alpha \tilde{\Pi}_{22}^{(\beta)}(\mathbf{k}_+, \mathbf{k}_-; \omega_s, \omega_s) + \frac{n_\mathrm{e} e^2}{mc} \delta_{\alpha\beta}. \tag{2}$$

Here $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2$; $\hbar\omega_s = (2s+1)\pi k_\mathrm{B} T$, $(s = 0, \pm 1, \pm 2, \ldots)$; n_e is the electron density in metal; the iterative indices mean summation. We define the vector $\vec{\tilde{\Pi}}_{22}(\mathbf{k}, \mathbf{k}'; \omega_s, \omega_{s'})$, according to [4] by supposing that ions in amorphous metal take one of numerous quasi-equilibrium positions (denoted with m), make small oscillations and interact with conduction electrons. We will describe this interaction by means of a local pseudopotential [4, 10]. The one-electron Green function in Nambu representation [1, 4] for the mth spatial ion configuration is

$$G^{(m)}(x, x') = \begin{pmatrix} g^{(m)}(x, x') & f^{(m)}(x, x') \\ f^{+(m)}(x, x') & g^{(m)}(x, x') \end{pmatrix}, \tag{3}$$

where

$$\begin{aligned} g^{(m)}(x, x') &= -\langle T_\tau(\hat{\psi}_\uparrow(x); \hat{\psi}_\uparrow^+(x'))\rangle^{(m)}, \\ f^{(m)}(x, x') &= -\langle T_\tau(\hat{\psi}_\uparrow(x); \hat{\psi}_\downarrow(x'))\rangle^{(m)}, \\ f^{+(m)}(x, x') &= -\langle T_\tau(\hat{\psi}_\downarrow^+(x); \hat{\psi}_\uparrow^+(x'))\rangle^{(m)}. \end{aligned} \tag{4}$$

Here the operators $\hat{\psi}_\alpha(x) \equiv \hat{\psi}_\alpha(\mathbf{r}, \tau)$ and $\hat{\psi}_\alpha^+(x) \equiv \hat{\psi}_\alpha^+(\mathbf{r}, \tau)$ with $0 \leqslant \tau \leqslant 1/k_\mathrm{B} T$ and spin α are the Heisenberg operators corresponding to the Schrödinger field operators $\hat{\psi}(\mathbf{r})$ and $\hat{\psi}^+(\mathbf{r})$, and the angular brackets denote the Gibbs averaging over all oscillating and electron degrees of freedom. The symbol T_τ is an ordering operator over variable τ. The properties of the one-electron Green function

$$G(x - x') = \overline{G^{(m)}(x, x')} = \frac{k_\mathrm{B} T}{(2\pi)^3} \sum_s \int \mathrm{d}^3 k \, \tilde{G}(\mathbf{k}, \omega_s) \exp(\mathrm{i}\mathbf{k}(\mathbf{r} - \mathbf{r}') - \mathrm{i}\omega(\tau - \tau'))$$

that is averaged over all configurations of quasi-equilibrium ion positions are described in detail in [4]. The choice of distribution function for configurational averaging depends on the specific character of the disorder model. Among the known models that describe the topological disorder of ions in amorphous metal, the most realistic is the model of a frozen liquid. This model is also used in the current work. That is why we will describe the structure of our amorphous system using a correlation function formalism, as in the theory of a liquid [10]. The Fourier image of electron Green functions for the weak electron–phonon bond takes the form [5, 6]

$$\begin{aligned} \tilde{g}(\mathbf{k}, \omega) &= \tilde{g}^*(-\mathbf{k}, -\omega) = -\frac{\mathrm{i}\hbar\omega\eta_\omega + \xi_\mathbf{k}}{\hbar^2\omega^2\eta_\omega^2 + \xi_\mathbf{k}^2 + \Delta^2\eta_\omega^2}, \\ \tilde{f}^+(\mathbf{k}, \omega) &= \tilde{f}^*(-\mathbf{k}, -\omega) = \frac{\Delta\eta_\omega}{\hbar^2\omega^2\eta_\omega^2 + \xi_\mathbf{k}^2 + \Delta^2\eta_\omega^2}, \end{aligned} \tag{5}$$

where Δ is the magnitude of the gap in the energy spectrum of the superconductor electron gas

$$\begin{aligned} &\xi_{\mathbf{k}} = \frac{\hbar^2}{2m}(k^2 - k_{\mathrm{F}}^2); \qquad \eta_\omega = 1 + \frac{1}{2\tau}\left(\omega^2 + \frac{\Delta^2}{\hbar^2}\right)^{-1/2}; \\ &\frac{1}{\tau} = \frac{mk_{\mathrm{F}}}{(2\pi)^3\hbar^3 v_0}\int |\tilde{w}(2k_{\mathrm{F}}\sin\Theta/2)|^2 a(2k_{\mathrm{F}}\sin\Theta/2)\,\mathrm{d}\Omega; \end{aligned} \tag{6}$$

$\tilde{w}(k)$ is the Fourier image of the screened pseudopotential of the electron–ion interaction; $a(k)$ is the structure factor of the amorphous metal, linked to the Fourier image of the binary distribution function by the relation $\tilde{S}(\mathbf{k}) = \frac{(2\pi)^3}{v_0}\delta(\mathbf{k}) + a(k)$; $v_0 = V/N$ is the volume per ion. The function

$$\begin{aligned} \vec{\Pi}_{22}(x - x_1, x' - x_1') = \frac{(k_{\mathrm{B}}T)^2}{(2\pi)^6}\sum_{s,s'}\int \mathrm{d}^3k\,\mathrm{d}^3k'\,\vec{\tilde{\Pi}}_{22}(\mathbf{k},\mathbf{k}';\omega_{\mathbf{s}},\omega_s') \\ \times\ \exp[\mathrm{i}\mathbf{k}(\mathbf{r} - \mathbf{r}_1) - \mathrm{i}\omega_s(\tau - \tau') - \mathrm{i}\mathbf{k}'(\mathbf{r}' - \mathbf{r}_1') + \mathrm{i}\omega_{s'}(\tau' - \tau_1')] \end{aligned}$$

is defined as

$$\vec{\Pi}_{22}(x - x_1; x' - x_1') = -\frac{\mathrm{i}}{2}\left(\frac{\partial}{\partial\mathbf{r}_1'} - \frac{\partial}{\partial\mathbf{r}}\right)\pi_{22}(x - x_1; x' - x_1'), \tag{7}$$

where

$$\pi_{22}(x - x_1; x' - x') = \overline{f^{+(m)}(x', x_1')f^{(m)}(x, x_1)} + \overline{g^{(m)}(x_1', x')g^{(m)}(x, x_1)}. \tag{8}$$

From (3) and (8), it follows that the function $\pi_{22}(x - x_1, x' - x_1')$ is an element of the matrix

$$\begin{aligned} \pi(x - x_1, x' - x_1') &= \overline{G^{(m)}(x'; x_1')G^{(m)}(x_1, x)} \\ &= \begin{pmatrix} \pi_{11}(x - x_1; x' - x_1') & \pi_{12}(x - x_1; x' - x_1') \\ \pi_{21}(x - x_1; x' - x_1') & \pi_{22}(x - x_1; x' - x_1') \end{pmatrix}. \end{aligned} \tag{9}$$

Dyson's equation for the matrix $\pi(x - x_1, x' - x_1')$ may be derived using the diagram technique of perturbation theory [1, 4]. It turns out that the equation can be written as

$$\begin{aligned} \pi(x - x_1, x' - x_1') &= G(x' - x_1')G(x_1 - x) \\ &+ \int \mathrm{d}^4y_1\,\mathrm{d}^4y_2\,\mathrm{d}^4y_3\,\mathrm{d}^4y_4\,G(x' - y_1)G(y_3 - x)K(y_1y_2y_3y_4) \\ &\times \pi(y_4 - x_1; y_2 - x_1') \end{aligned} \tag{10}$$

where the kernel $K(y_1y_2y_3y_4)$ corresponds to a specific set of Feynman diagrams. If we limit our attention to a quadratic approximation of the pseudopotential, then the expression for the kernel $K(y_1y_2y_3y_4)$ takes the form

$$K(y_1y_2y_3y_4) = \frac{1}{(2\pi)^3 v_0}\int \mathrm{d}^3q\,|\tilde{w}(q)|^2 a(q)\exp(\mathrm{i}\mathbf{q}(\mathbf{r}_{y_1} - \mathbf{r}_{y_3}))\delta(y_1 - y_2)\delta(y_3 - y_4).$$

In this approximation equation (10) transforms to

$$\begin{aligned} \pi(x - x_1, x' - x_1') = G(x' - x_1')G(x_1 - x) + \int \mathrm{d}^4y_1\int \mathrm{d}^4y_2\,G(x' - y_1)G(y_2 - x) \\ \times \int \frac{\mathrm{d}^3q}{(2\pi)^3}\frac{|\tilde{w}(q)|^2}{v_0}a(q)\exp[\mathrm{i}\mathbf{q}(\mathbf{r}_{y_1} - \mathbf{r}_{y_2})]\pi(y_2 - x_1; y_1 - x_1'). \end{aligned} \tag{11}$$

Here $x = (\mathbf{r}, \tau)$; $y = (\mathbf{r}_y, \tau_y)$.

If we apply to equation (11) with operator $-\frac{\mathrm{i}}{2}(\frac{\partial}{\partial \mathbf{r}_1'} - \frac{\partial}{\partial \mathbf{r}})$ and write the resulting equation in a Fourier form, we can easily obtain the equation for the Fourier image of the vector $\vec{\Pi}(x - x_1; x' - x_1')$:

$$\vec{\tilde{\Pi}}(\mathbf{k}_+, \mathbf{k}_-; \omega, \omega) = \mathbf{k}\tilde{G}(\mathbf{k}_-; \omega)\tilde{G}(\mathbf{k}_+; \omega) + \tilde{G}(\mathbf{k}_+; \omega)\tilde{G}(\mathbf{k}_-; \omega) \times \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \frac{|\tilde{w}(|\mathbf{k} - \mathbf{k}'|)|^2}{v_0} a(|\mathbf{k} - \mathbf{k}'|)\vec{\tilde{\Pi}}(\mathbf{k}_+', \mathbf{k}_-'; \omega, \omega). \tag{12}$$

Using the explicit form of the vector $\vec{\tilde{\Pi}}(\mathbf{k}_+, \mathbf{k}_-; \omega, \omega')$ and $\tilde{G}(\mathbf{k}, \omega)$ (see (3), (7) and (9)), we will find the system of equations for all elements of the vector $\vec{\tilde{\Pi}}$. The last two equations are

$$\begin{aligned} \vec{\tilde{\Pi}}_{12}(\mathbf{k}_+, \mathbf{k}_-; \omega, \omega) &= \mathbf{k}A_{12}(\mathbf{k}_+, \mathbf{k}_-, \omega)[1 + \Lambda_{22}(\omega)] + \mathbf{k}A_{11}(\mathbf{k}_+, \mathbf{k}_-, \omega)\Lambda_{12}(\omega), \\ \vec{\tilde{\Pi}}_{22}(\mathbf{k}_+, \mathbf{k}_-; \omega, \omega) &= \mathbf{k}A_{22}(\mathbf{k}_+, \mathbf{k}_-, \omega)[1 + \Lambda_{22}(\omega)] + \mathbf{k}A_{21}(\mathbf{k}_+, \mathbf{k}_-, \omega)\Lambda_{12}(\omega), \end{aligned} \tag{13}$$

where

$$\mathbf{k}\Lambda_{ij}(\omega) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \frac{|\tilde{w}(|\mathbf{k} - \mathbf{k}'|)|}{v_0} a(|\mathbf{k} - \mathbf{k}'|)\vec{\tilde{\Pi}}_{ij}(\mathbf{k}_+', \mathbf{k}_-', \omega) \tag{14}$$

$$\begin{aligned} A_{12}(\mathbf{k}_+, \mathbf{k}_-, \omega) &= A_{21}^*(\mathbf{k}_+, \mathbf{k}_-, \omega) = \tilde{f}(\mathbf{k}_+, \omega)\tilde{g}(\mathbf{k}_-, \omega) + \tilde{g}(-\mathbf{k}_+, -\omega)\tilde{f}(\mathbf{k}_+, \omega) \\ A_{22}(\mathbf{k}_+, \mathbf{k}_-, \omega) &= A_{11}^*(\mathbf{k}_+, \mathbf{k}_-, \omega) = \tilde{f}(\mathbf{k}_+, \omega)\tilde{f}^+(\mathbf{k}_-, \omega) + \tilde{g}(-\mathbf{k}_+, -\omega)\tilde{g}(\mathbf{k}_-, -\omega). \end{aligned} \tag{15}$$

As long as $|\mathbf{k}| \sim |\mathbf{k}_\mathrm{F}|$, we can assume that the function $\Lambda_{ij}(\omega)$ is independent of $|\mathbf{k}|$.

Multiplying both sides of equation (13) by $\frac{|\tilde{w}(|\mathbf{k}-\mathbf{k}'|)|^2}{v_0} a(|\mathbf{k} - \mathbf{k}'|)$ and taking the integral over $\mathbf{k}'$, we get the system of equations for the functions $\Lambda_{ij}(\omega)$:

$$\begin{aligned} \mathbf{k}\Lambda_{12}(\omega) &= \overline{\mathbf{k}A_{12}} \cdot [1 + \Lambda_{22}(\omega)] + \overline{\mathbf{k}A_{11}} \cdot \Lambda_{12}(\omega), \\ \mathbf{k}\Lambda_{22}(\omega) &= \overline{\mathbf{k}A_{22}} \cdot [1 + \Lambda_{22}(\omega)] + \overline{\mathbf{k}A_{21}} \cdot \Lambda_{21}(\omega), \end{aligned} \tag{16}$$

where

$$\overline{\mathbf{k}A_{ij}} = \int \frac{\mathrm{d}^3k'}{(2\pi)^3} \frac{|\tilde{w}(|\mathbf{k} - \mathbf{k}'|)|^2}{v_0} a(|\mathbf{k} - \mathbf{k}'|)\mathbf{k}'A_{ij}(\mathbf{k}_+', \mathbf{k}_-', \omega). \tag{17}$$

In relationships (1) and (2) the quantity $|\mathbf{q}| \sim 1/\delta_\mathrm{L}$, where δ_L is the penetration depth of the electromagnetic field into the superconductor, and $|\mathbf{k}| \sim 1/l_\mathrm{e}$ is the free electron path in the metal. From the assumption that in the amorphous metal $l_\mathrm{e} \ll \delta_\mathrm{L}$, it follows that $|\mathbf{k}| \gg |\mathbf{q}|$. Thus in expressions (13), (14), (15) and (17) we can put $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2 \approx \mathbf{k}$.

Taking into account that $|\mathbf{k}| \sim |\mathbf{k}'| \sim k_\mathrm{F}$ in (17) and using the relation [1, 2]:

$$\int \frac{\mathrm{d}^3k'}{(2\pi)^3} f(\mathbf{k}') \approx \frac{mk_\mathrm{F}}{(2\pi)^3\hbar^2} \int_{-\infty}^{+\infty} \mathrm{d}\xi_\mathbf{k}' \int \mathrm{d}\Omega' f(k_\mathrm{F}, \Omega', \xi_\mathbf{k}'). \tag{18}$$

Then the quantity $\overline{\mathbf{k}A_{ij}}$ is written as

$$\overline{\mathbf{k}A_{ij}} = \mathbf{k} \cdot \overline{A_{ij}} = \mathbf{k}\frac{\hbar}{2\pi\tau_1}\tilde{A}_{ij}, \tag{19}$$

where

$$\frac{1}{\tau_1} = \frac{mk_\mathrm{F}}{(2\pi)^2\hbar^3} \int \frac{|\tilde{w}(2k_\mathrm{F}\sin\Theta/2)|^2}{v_0} a(2k\sin(\Theta/2))\cos\Theta \cdot \mathrm{d}\Omega \tag{20}$$

$$\tilde{A}_{ij} = \int_{-\infty}^{+\infty} A_{ij}(\mathbf{k}, \mathbf{k}, \omega)\,\mathrm{d}\xi_\mathbf{k}. \tag{21}$$

If the electron–phonon bond is weak, we can use (5) for the calculation of $\tilde{A}_{ij}$.

$$\begin{aligned}\overline{A_{11}} &= \overline{A_{22}} = \frac{\hbar}{2\tau_1}\cdot\frac{\Delta^2}{\eta_\omega(\hbar^2\omega^2+\Delta^2)^{3/2}},\\ \overline{A_{12}} &= \overline{A_{21}} = 0.\end{aligned} \tag{22}$$

Putting these expressions for $\overline{A_{ij}}$ into the equations (16), we will find

$$\begin{aligned}&\Lambda_{12}(\omega) = 0,\\ &1+\Lambda_{22}(\omega) = 1/(1-\overline{A_{11}}).\end{aligned} \tag{23}$$

From this, the unknown function $\vec{\tilde{\Pi}}_{22}$, according to (13), is

$$\vec{\tilde{\Pi}}_{22}(\mathbf{k}_+,\mathbf{k}_-;\omega,\omega) \approx \vec{\tilde{\Pi}}(\mathbf{k},\mathbf{k};\omega,\omega) = \mathbf{k}A_{22}(\mathbf{k},\mathbf{k},\omega)/(1-\overline{A_{11}}). \tag{24}$$

Putting the obtained expression for $\vec{\tilde{\Pi}}_{22}$ into relation (2), and using it to integrate relation (18), we can find the next expression for the core $Q_{\alpha\beta}(\mathbf{q})$:

$$Q_{\alpha\beta}(\mathbf{q}) = \frac{n_e e^2}{mc}\delta_{\alpha\beta}\left\{1+k_B T\sum_{s=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}\xi_{\mathbf{k}}\,\frac{A_{22}(\mathbf{k},\mathbf{k},\omega_s)}{1-\overline{A_{11}}}\right\}. \tag{25}$$

We can calculate the second term using the relations (5), (15) and (22) by taking into account that the second term is the following limit [1]:

$$\begin{aligned}k_B T\sum_s\int_{-\infty}^{+\infty}\mathrm{d}\xi_{\mathbf{k}}\,\frac{A_{22}(\mathbf{k},\mathbf{k},\omega_s)}{1-\overline{A_{11}}} = \lim_{\alpha\to 0}\Bigg\{k_B T\sum_s\left[1-\frac{\hbar}{2\tau_1}\frac{\Delta^2}{\eta_\omega(\hbar^2\omega_s^2+\Delta^2)^{3/2}}\right]\\ \times\int_{-\infty}^{+\infty}\mathrm{d}\xi_{\mathbf{k}}\,\cos(\alpha\xi_{\mathbf{k}})\frac{-\hbar^2\omega_s^2\eta_\omega^2+\Delta^2\eta_\omega^2+\xi_{\mathbf{k}}^2}{[\hbar^2\omega_s^2\eta_\omega^2+\Delta^2\eta_\omega^2+\xi_{\mathbf{k}}^2]^2}\Bigg\}.\end{aligned}$$

Using the calculus of residuals, it is easy to show that

$$\begin{aligned}&k_B T\sum_s\int_{-\infty}^{+\infty}\mathrm{d}\xi_{\mathbf{k}}\,\frac{A_{22}(\mathbf{k},\mathbf{k},\omega_s)}{1-\overline{A_{11}}}\\ &\qquad = -1+2\pi k_B T\sum_{s=0}^{\infty}\frac{\Delta^2}{(\hbar^2\omega_s^2+\Delta^2)\left[\sqrt{\hbar^2\omega_s^2+\Delta^2}+\hbar/(2\tau_{\mathrm{tr}})\right]},\end{aligned} \tag{26}$$

where

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{1}{\tau}-\frac{1}{\tau_1} = \frac{mk_F}{(2\pi)^2\hbar^3}\int\frac{|\tilde{w}(2k_F\sin(\Theta/2))|}{v_0}a(2k_F\sin(\Theta/2))(1-\cos\Theta)\,\mathrm{d}\Omega. \tag{27}$$

The quantity τ_{tr} is called the 'transport' time between the impact of electrons with disordered ions of metal and it defines the residual electrical resistance of amorphous metals:

$$\rho_0 = \frac{m}{n_e e^2}\cdot\frac{1}{\tau_{\mathrm{tr}}}. \tag{28}$$

The expression (27) was first obtained by Ziman for disordered systems [10, 11]. Substituting (26) in (25) one can find the expression for the kernel $Q_{\alpha\beta}$:

$$Q_{\alpha\beta}(q) = Q\delta_{\alpha\beta}, \tag{29}$$

where

$$Q = \frac{n_e e^2}{mc}2\pi k_B T\sum_{s>0}\frac{\Delta^2}{(\hbar^2\omega_s^2+\Delta^2)\left[\sqrt{\hbar^2\omega_s^2+\Delta^2}+\hbar^2/(2\tau_{\mathrm{tr}})\right]}. \tag{30}$$

This relation had been obtained first by Abrikosov and Gorkov during the investigations of electromagnetic properties of crystalline alloys [1, 6–8]. In their case, the 'transport' time τ_{tr} was highly dependent on the impurity density inside the superconductor (i.e. when the concentration approached zero $1/\tau_{tr} \to 0$). In our case τ_{tr} depends on the structure of the metal and is finite for pure amorphous metals.

The penetration depth δ_L of the magnetic field into the superconductor is defined by [2, 3]

$$\delta_L = \sqrt{\frac{c}{4\pi Q}}. \tag{31}$$

If $1/\tau_{tr} \to 0$ (i.e. during the transition from amorphous metal to crystal) this formula turns into a usual London expression:

$$\delta_L = \sqrt{\frac{mc^2}{4\pi n_s e^2}}, \tag{32}$$

where

$$n_s = n_e \cdot 2\pi k_B T \sum_s \frac{\Delta^2}{(\hbar^2\omega_s^2 + \Delta^2)^{3/2}}$$

is the density of superconducting electrons (when $T \to 0$ $n_s \to n_e$).

In the reverse case, when $\frac{t}{2\tau_{tr}} \gg \Delta$, or $l_e = v_F\tau_{tr} \ll \frac{2\hbar v_F}{\Delta} = 2\xi_0$, it is possible in (30) to neglect the term under the square root in the denominator. Then the remaining series can be added, and for Q we will get the expression

$$Q = \frac{n_e e^2 \tau_{tr}}{m} \frac{\pi}{c\hbar} \Delta \cdot \tanh\left(\frac{\Delta}{2k_B T}\right).$$

From this

$$\delta_L(T) = \sqrt{\frac{mc^2\hbar}{4\pi^2 n_e e^2 \tau_{tr} \Delta \tanh(\Delta/2k_B T)}}. \tag{33}$$

If we know $\delta_L(T)$ then it is easy to define the Ginzburg–Landau parameter $\chi(T)$. Using the well-known relation $\chi(T) = 2^{3/2}(\hbar c)^{-1} H_c(T)\delta^2(T)$, where $H_c(T)$ is the critical magnetic field that destroys the superconductivity, we can find the following expression for the Ginzburg–Landau parameter [2, 3]:

$$\chi(T) = \chi(T_c)B(T), \tag{34}$$

where

$$\chi(T_c) = 0.72\delta_L(0)/l_e, \tag{35}$$

and more nice calculations of $\delta_L(T)$ show that the function $B(T)$ decreases from 1.2 to 1 (see [2, 9]) in the limit of $(0, T_c)$.

If for simple amorphous metals we use the model of a 'frozen' liquid, then for the numerical calculations of $l_e = v_F\tau_{tr}$ it is possible to use the hard sphere model and the pseudopotential of Ashcroft [4]. In the following table, the calculation results for the quantity $\delta_L(0)/l_e$ and the Ginzburg–Landau parameter $\chi(T)$ are given for almost all simple metals. One can see that the quantity $\chi(T_c)$ is greater than $1/\sqrt{2} = 0.7071$ for all metals seen in table 1 (except Al, Sn). This means that almost all simple metals in a superconducting state are type II superconductors. Having the value of $\chi(T)$ and using the general theory [2, 3], it is possible to calculate the lower value of the critical field $H_{c1}(T)$, defined as the transition of matter from a homogeneous superconducting state into a mixed one. Similarly it is possible to find the higher

Table 1.

Metal	$\delta_L(0)/l_e$	$\chi(T_c)$	$H_{c1}(T_c)/H_c(T_c)$	$H_{c2}/H_c(T_c)$
Li	4.02	2.89	0.39	4.09
Cs	4.97	3.58	0.31	5.06
Zn	2.6	1.87	0.57	2.65
Al	0.73	0.53	1	1
Ga	5.83	4.2	0.25	5.93
In	1.09	0.79	0.98	1.11
Sn	0.94	0.675	1	1
Pb	1.69	1.22	0.77	1.72
Bi	1.54	1.11	0.83	1.57

value $H_{c1}(T)$, i.e. the highest field at which the mixed superconducting state exists. The lower critical field is defined by the expression $H_{c1} = f(\chi)H_c$, where $f(\chi) \to 1$ when $\chi \to 1/\sqrt{2}$ and $f(\chi) \approx \frac{\ln\chi + 0.081}{\sqrt{2}\chi}$ when $\chi \gg 1$, and the higher critical field takes a value $H_{c2} = \sqrt{2}\chi H_c$. In the table the values H_{c1}/H_c and H_{c2}/H_c when $T = T_c$ are collected. If the field takes the values $H_{c1} < H < H_{c2}$ then the superconductor stays in the mixed state, i.e. consisting of ordered domains of superconducting and normal phases. The mixed state might be described as a lattice structure that consists of domains (vortex filament), in which the ordering factor changes quickly with domain sizes, while the microscopic field stays almost invariable [2, 3]. When the electric current is normal to the direction of the applied magnetic field, the vortex filament lattice feels the influence of the Lorentz force. In the absence of any other forces the lattice evolves into a viscous flow. The flow is connected with energy dissipation; therefore the specimen is no longer superconducting. Under these conditions, the critical current density $I_c = 0$ when $H > H_{c1}$. However, the situation described is applicable only to homogeneous type II superconductors. In heterogeneous materials, additional interactions of the vortex filament lattice and sample inhomogeneities exist. Under favourable conditions these interactions (pinning) lead to resistance for movement or stopping of movement of the vortex lattice, thus $I_c > 0$ when $H > H_{c1}$. In this sense an amorphous metal is a inhomogeneous material, since at low temperatures it holds quite a large density of low-temperature excitations ('two-level systems') and impregnations of finely dispersed crystal phase (the amorphous state is a thermodynamically unstable system), which can interact with vortex filaments. That is why a strong pinning can occur in amorphous superconductors.

References

[1] Abrikosov A A, Gor'kov L P and Dzialoshinskii I E 1962 *Methods of Quantum Field Theory in Statistical Physics* (Moscow: Fizmatgiz)
[2] Abrikosov A A 1987 *Fundamentals of Metals Theory* (Moscow: Nauka)
[3] Lifshiz E M and Pitaevskii L P 1978 *Statistical Physics* part 2 (Moscow: Nauka)
[4] Krasny Y P and Kovalenko N P 1990 *Physica* B **162** 128
[5] Güntherodt H-J and Beck H (ed) 1981 *Glassy Metals I* (Berlin: Springer)
[6] Abrikosov A A and Gor'kov L P 1958 *Zh. Eksp. Teor. Fiz.* **35** 1558
[7] Abrikosov A A and Gor'kov L P 1958 *Zh. Eksp. Teor. Fiz.* **36** 319
[8] Gor'kov L P 1959 *Zh. Eksp. Teor. Fiz.* **37** 1407
[9] Heffand E and Werthamer N R 1966 *Phys. Rev.* **147** 288
[10] Kovalenko N P, Krasny Yu P and Trigger S A 1990 *Statistical Theory of Liquid Metals* (Moscow: Nauka)
[11] Ziman J M 1961 *Phil. Mag.* **6** 1013